

## Learning the Sudoku - easy ones first

Complete this 8x8 Sudoku puzzle by placing a number from 1 to 8 in each empty square. Place the numbers so that no number repeats in any row, column or bold lined box.

			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1							6
	7	2			3	1	
	5	8			2	3	
			7	4			

How do we solve this?

Let us begin by trying to fill the boxes one by one.

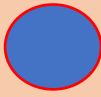
2 is not present in the Third Box on Right Side.

Where to put it?

			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1							6
	7	2			3	1	
	5	8			2	3	
			7	4			

We draw vertical line along 2 to show in which boxes no other 2 can appear.

Which other boxes 2 cannot appear?

			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1							6
	7	2			3	1	
	5	8			2	3	
			7	4			

Now we are left with only one empty box where 2 must appear!

So let us fill it in.

Which other boxes 2 cannot appear?

			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
			7	4			

### What next?

There are lot of 7s on the left side columns, but none in Top Left Box. Seems easy to fill in a 7 there!

			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
			7	4			

### How do we solve this?

Let us begin by trying to fill the boxes one by one.

After showing all the boxes where 7 cannot go, we have only one empty Box.

We shall fill in 7 there.

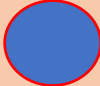
Now we have put the 7 in the right box

7			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
			7	4			

What do we search for next?

### What do we search for next?

Having put the 7 in the right box, we see on the left half, three columns have 1s running through them. Only one Column does not have 1. And that empty box is in bottom Left. Do you notice that?

7			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
			7	4			

So 1 can go in that empty box!

After filling that 1, how will it look now?

After filling that 1, how will it look now?

7			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

What do we search for next?

On the right half, you see two 3s in two columns, so let us have lines run through them.



### What do we search for next?

On the right half, you see two 3s in two columns, so let us have lines run through them.

We also have a 3 in the second row, so let us clear that row as well.

7			1	2			
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

Now can you spot the only space in the Top Right Box where 3 can go in?

Let us fill that space?

After filling that 3, how will it look now?

7			1	2			3
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

What do we search for next?

Again in Top Left Box, can you see where you can put a 5?

In Top Left Box, can you see where you can put a 5?

7			1	2			3
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

You must have spotted already!

Having found the space for the 5, let us put it?

7		5	1	2			3
	2	3			5	7	
	1	7			6	4	
3							2
1						2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

### Look at Columns now

There are other ways to fill the Sudoku. Let us look at Columns now.

The third Column has 6 spaces already filled in, so let us focus on that.

What do we see?

We have two empty spaces only. But what can go in there?

This column already has 1, 2, 3, 5, 7, 8

So only 4 and 6 are remaining.

In the two empty spaces, 4 must go into one place and 6 in another.

Where 4 and where 6?

Where 4 and where 6?

7		5	1	2			3
	2	3			5	7	
	1	7			6	4	
3							2
1		X				2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

Notice that 6 cannot go into 5<sup>th</sup> Row because one 6 is already there in 5<sup>th</sup> Row.

Hence 6 must go into the only other space that is empty.

And 4 must be in this space where 6 cannot come in.

We have now filled 4 and 6 in the right spaces in the Third Column.

7		5	1	2			3
	2	3			5	7	
	1	7			6	4	
3		6					2
1		4				2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

**What are other ways to progress?**

Let us look at empty spaces which are Together, in the same Box, and see if there is some number that cannot go in there. Let us see how it works.

## Let us look at empty spaces which are Together

We mark one some area in the 4<sup>th</sup> row

Also notice that 3 cannot go in there.

7		5	1	2			3
	2	3			5	7	
	1	7			6	4	
3		6					2
1		4				2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

And 3 cannot come in the last column because one 3 is already in the last column.

This means there is only one empty space for 3 in that box.

Let us fill it in.

Having filled the 3,

Let us look at empty spaces which are Together

In the shaded spaces, you see where 4 cannot go in.

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	
	5	8			2	3	
		1	7	4			

So we are left with one space (shaded yellow) where 4 can go in.

Look carefully.

So let us fill in that 4.



We have now filled the 4,

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	4
	5	8			2	3	
		1	7	4			

## Another case of contiguous blank spaces

Last row, there are 3 such spaces together (see shaded area)

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	4
	5	8			2	3	
		1	7	4			

1 and 7 are in the box on the left, so 1 and 7 cannot go in here.

So we can have 1 and 7 only on the row above. And there are only two empty spaces (shaded yellow), so both 1 and 7 will go in there but we don't know which will go where.

## 1 and 7 are pairs

Keep the pair reserved in the pair of spaces, but we don't know which goes where.

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	1 or 7
2 or 3	2 or 3	1	7	4			

Now look at the 7<sup>th</sup> Row and Bottom Left Box.

You will see that 2 and 3 cannot go into the 7<sup>th</sup> Row, Bottom Left as there are in the Same Row, other Box.

So 2 and 3 must go into the 8<sup>th</sup> Row. And we write that Pair in the Pair of spaces.

This time it is easy to see where 2 should go and where 3 should go

## 2 and 3 are pairs

Keep the pair reserved in the pair of spaces, but this time it is easy to see where 2 should go and where 3 should go

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	1 or 7
2 or 3	2 or 3	1	7	4			

In the first column, 3 is there, and in the second column 2 is there, so we know how to fill in this pair of 2 and 3.

Fill in 2 and 3 in the last row

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	1 or 7
2	3	1	7	4			

Having filled in 2 and 3, we see in last row, there are three spaces remaining (shaded blue)

Since we have 1, 2, 3, 4, 7 in the last row already, we have to fill in 5, 6, 8 in the last row in the blue spaces.

Fill in 5, 6, 8 in the blue spaces in the last row

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	1 or 7
2	3	1	7	4			

How to fill in 5, 6, 8?

See that in the 6<sup>th</sup> column, we have 5 and 6 (dotted green line). So we can have only 8 in that space.

We have 6 in the last column (dotted blue line), so 6 cannot be there, and 8 is already filled in, so it must be 5.

And 6 will come in between 8 and 5. (6 could not have come in any other space as those columns have 6.

Filling in 5, 6, 8 in the last row

7		5	1	2			3
	2	3			5	7	
	1	7		3	6	4	
3		6					2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	1 or 7
2	3	1	7	4	8	6	5

Now go to 7<sup>th</sup> Column where 6 spaces are filled, and only two remain.

5 and 8 are remaining to be filled in the spaces marked yellow.

In the yellow in First Row, you cannot have 5 as we already have another 5 in the First Row (shaded blue), so it must be filled 8.

In the other Yellow, therefore it must be 5.

Filled in 5, 8 in the yellow spaces in 7th row

7		5	1	2		8	3
	2	3			5	7	
	1	7		3	6	4	
3		6				5	2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	1 or 7
2	3	1	7	4	8	6	5



## How much have we done so far?

Now two columns and one row are full ! So we are making progress 😊

Now for the Home Stretch.

(What does Home Stretch mean?)

## What next?

The last column has only 3 empty spaces. Let us go after that!

Since 2, 3, 4, 5, 6 are filled, we have to find where to put in 1, 7, 8

Third Row has 1 and 7 (yellow shaded), so in third row, last column, we can have only 8.

Now 1 and 7 are remaining. Second Row already has 7 (blue shaded), so Second row, last column, will be 1.

This means the remaining space, i.e. 7th row, last column must be 7

7		5	1	2		8	3
	2	3			5	7	
	1	7		3	6	4	
3		6				5	2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	1 or 7
2	3	1	7	4	8	6	5

So how will the new matrix look?

### So how will the new matrix look?

- Third Row has 1 and 7 (yellow shaded), so in **third row, last column**, we can have only 8.
- Now 1 and 7 are remaining.
- Second Row already has 7 (blue shaded), so **Second row, last column**, will be 1.
- This means the remaining space, i.e. **7th row, last column** must be 7

7		5	1	2		8	3
	2	3			5	7	1
	1	7		3	6	4	8
3		6				5	2
1		4				2	6
	7	2			3	1	4
	5	8		1 or 7	2	3	7
2	3	1	7	4	8	6	5

This also means the space in 5<sup>th</sup> Column, 7<sup>th</sup> Row, whose value is 1 or 7 (shaded Brown), can now be fixed as 1. (As 7 is taken already, in the last column.)

What next?

Let us next consider the 6<sup>th</sup> Column – it has 3 empty spaces and 1, 4, 7 not yet filled.

## What next?

Let us next consider the 6<sup>th</sup> Column – it has 3 empty spaces and 1, 4, 7 not yet filled.

7		5	1	2		8	3
	2	3			5	7	1
	1	7		3	6	4	8
3		6				5	2
1		4				2	6
	7	2			3	1	4
	5	8		1	2	3	7
2	3	1	7	4	8	6	5

The three empty spaces must have 1, 4, 7 inside them.

5<sup>th</sup> Row (yellow shade) already has 1 and 4. So 5<sup>th</sup> Row in 6<sup>th</sup> Column must be 7

Remaining to be filled: 1 and 4

1<sup>st</sup> Row (Top row) , 4<sup>th</sup> Column has 1 (blue shade). So Top Row, 6<sup>th</sup> Column has to be 4

This means, 4<sup>th</sup> Row, 6<sup>th</sup> Column should be 1

## How does the new matrix look now

The 6<sup>th</sup> Column is now filled.

7	6	5	1	2	4	8	3
	2	3		6	5	7	1
	1	7		3	6	4	8
3		6		7	1	5	2
1		4		5 or 8	7	2	6
	7	2		5 or 8	3	1	4
	5	8		1	2	3	7
2	3	1	7	4	8	6	5

## Let us fill some more!

The Top Right Box has 7 numbers filled, and only 6 is not filled. So the empty space (green shade) must have 6.

The Box below, i.e. 2<sup>nd</sup> Box from Top, on the Right Side, also has only one Empty space (blue shade) and one number is missing, i.e. 7. So 7 will get into that space.

Now look at the Box below, we will see two empty spaces in the 5<sup>th</sup> Column (violet shade). The two missing numbers are 5 and 8. Let us put that in the space.

The Top Row has only one empty space (yellow shaded) and only 6 is missing in that row. So fill in 6.

## How does the new matrix look now

Most of the right half is filled.

Two rows are also filled.

7	6	5	1	2	4	8	3
	2	3		6	5	7	1
5	1	7	2	3	6	4	8
3	4	6		7	1	5	2
1	8	4		5 or 8	7	2	6
	7	2		5 or 8	3	1	4
	5	8		1	2	3	7
2	3	1	7	4	8	6	5

## Let us fill some more!

**3<sup>rd</sup> Row – two spaces free** and we need to fill in 2 and 5

- But first column already has a 2 (yellow shade). So 1<sup>st</sup> Column in 3<sup>rd</sup> Row cannot be 2, it has to be 5. This means the other empty space has to be 2.

**2<sup>nd</sup> Column – two spaces free** (blue shade) and we need to fill in 4 and 8 in the empty spaces (blue shade)

5<sup>th</sup> Row has a 4, so we need to fill in 8 in our space in 5<sup>th</sup> Row.

This leaves 4 to be filled in the other empty space.

**Now note the brown shaded spaces in 5<sup>th</sup> Column with value 5 or 8.**

Since 5<sup>th</sup> Row already has an 8 (the blue space we filled in just now), the free space in 5<sup>th</sup> Row, 5<sup>th</sup> Column must be 5. That leaves the other brown space to be 8.

How does the new matrix look now?

Now almost everything looks filled. Can you not do the rest on your own??

7	6	5	1	2	4	8	3
	2	3		6	5	7	1
5	1	7	2	3	6	4	8
3	4	6		7	1	5	2
1	8	4		5	7	2	6
	7	2		8	3	1	4
	5	8		1	2	3	7
2	3	1	7	4	8	6	5

And after you have tried on your own ...

Go to the next page

## Towards completion

4<sup>th</sup> Row, one empty space is left (yellow), and 8 is missing – so it should be 8

5<sup>th</sup> Row, one empty space is left (blue), and 3 is missing – so it should be 3

2<sup>nd</sup> Row, two empty spaces are left, and 4 and 8 are missing

- The 4<sup>th</sup> Column already has a 8 (the yellow space), so 2<sup>nd</sup> Row 4<sup>th</sup> Column (light green) cannot be 8, must be 4. Hence 2<sup>nd</sup> Row 1<sup>st</sup> Column (dark green) must be 8.

7	6	5	1	2	4	8	3
8	2	3	4	6	5	7	1
5	1	7	2	3	6	4	8
3	4	6	8	7	1	5	2
1	8	4	3	5	7	2	6
6	7	2		8	3	1	4
4	5	8		1	2	3	7
2	3	1	7	4	8	6	5

Now 1<sup>st</sup> Column – two empty spaces (Orange) and 4 and 6 are remaining

The 6<sup>th</sup> Row already has a 4 (violet), so we must have 6 in the 6<sup>th</sup> Row, 1<sup>st</sup> Column

So 4 remains for the other blank (Orange) space.

Now filling the last two should be trivial !!

## The final answer

7	6	5	1	2	4	8	3
8	2	3	4	6	5	7	1
5	1	7	2	3	6	4	8
3	4	6	8	7	1	5	2
1	8	4	3	5	7	2	6
6	7	2	5	8	3	1	4
4	5	8	6	1	2	3	7
2	3	1	7	4	8	6	5